Learning from Observations

Chapter 18, Sections 1–3

Outline

◊ Inductive learning

◊ Decision tree learning

Learning a model from data

Can involve estimating parameters and/or learning structure of model

Example of parameter estimation: estimating conditional probabilities in Bayesian networks

Practical application: naive Bayes document classifier

Inductive learning (a.k.a. Science)

Simplest form: learn a function from examples (supervised learning)

f is the target function

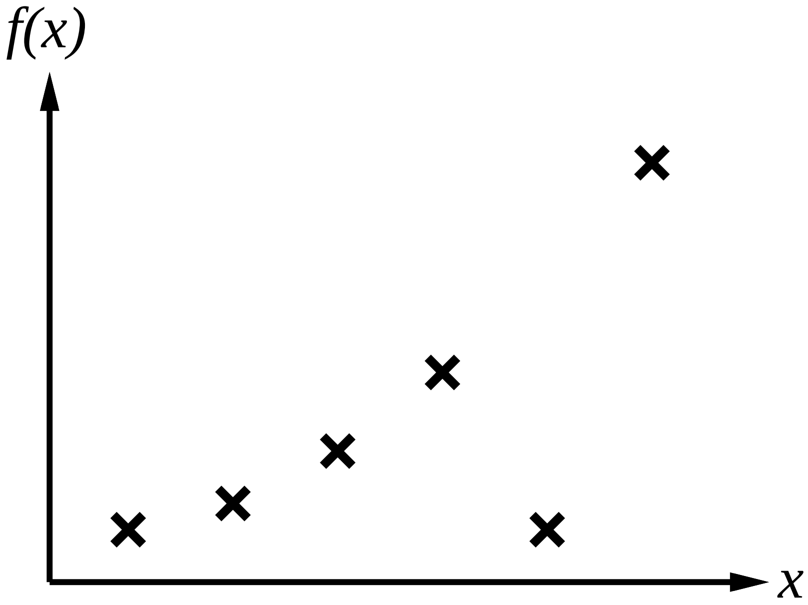
An example is a pair x, $f(x)$, e.g., $\begin{array}{c|c|c} O & O & X \\ \hline & X & \\ \hline X & & \end{array} \;,\;\; +1$

Problem: find a(n) hypothesis h
such that $h \approx f$
given a training set of examples

Inductive learning method

Construct/adjust h to agree with f on training set
(h is consistent if it agrees with f on all examples)

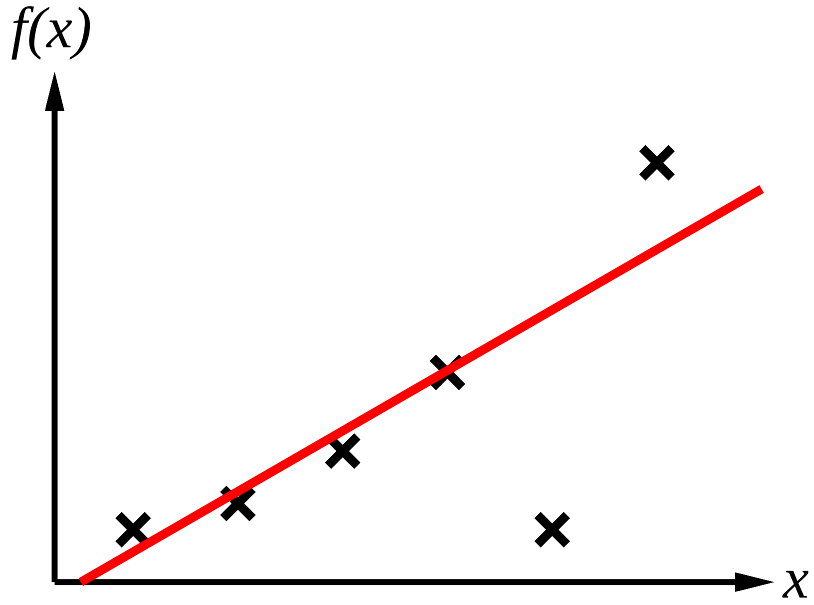
E.g., curve fitting:



Inductive learning method

Construct/adjust h to agree with f on training set
(h is consistent if it agrees with f on all examples)

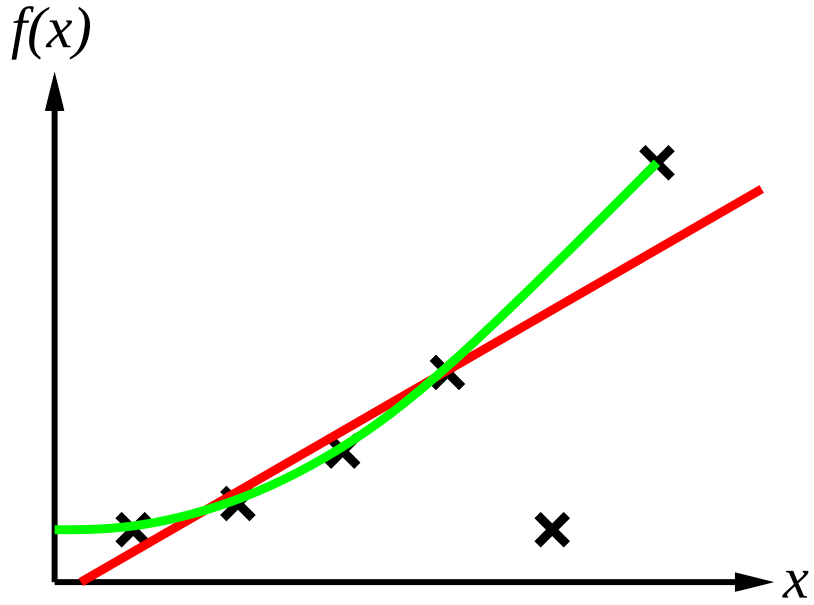
E.g., curve fitting:



Inductive learning method

Construct/adjust h to agree with f on training set
(h is consistent if it agrees with f on all examples)

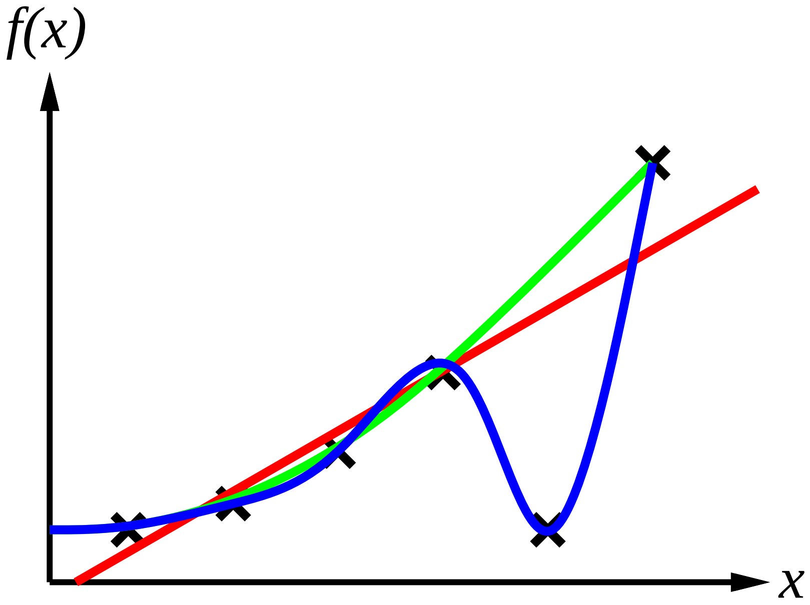
E.g., curve fitting:



Inductive learning method

Construct/adjust h to agree with f on training set
(h is consistent if it agrees with f on all examples)

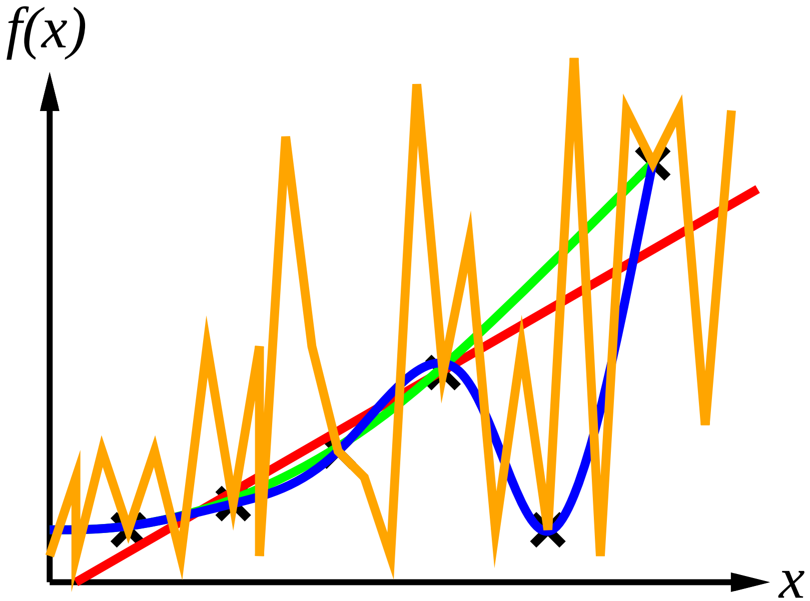
E.g., curve fitting:



Inductive learning method

Construct/adjust h to agree with f on training set
(h is consistent if it agrees with f on all examples)

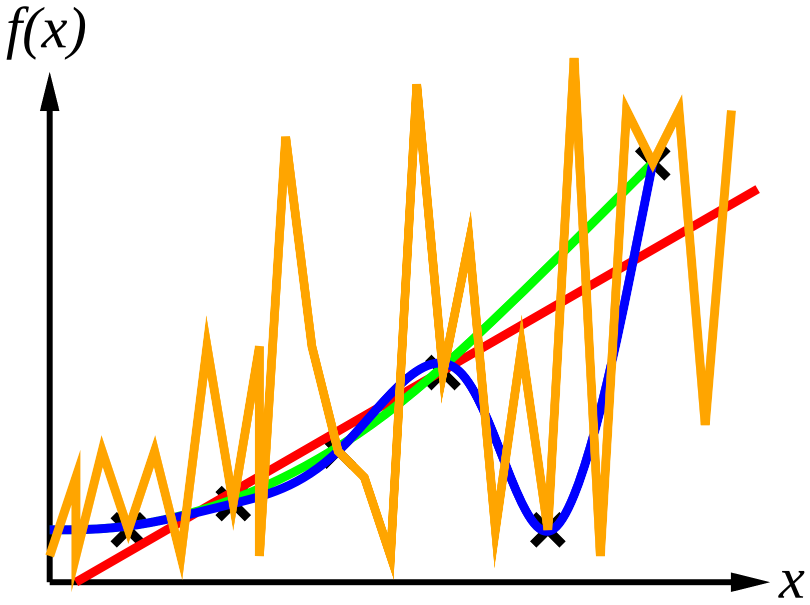
E.g., curve fitting:



Inductive learning method

Construct/adjust h to agree with f on training set
(h is consistent if it agrees with f on all examples)

E.g., curve fitting:



Ockham's razor: maximize a combination of consistency and simplicity

Attribute-based representations

Examples described by attribute values (Boolean, discrete, continuous, etc.)
E.g., situations where I will/won't wait for a table:

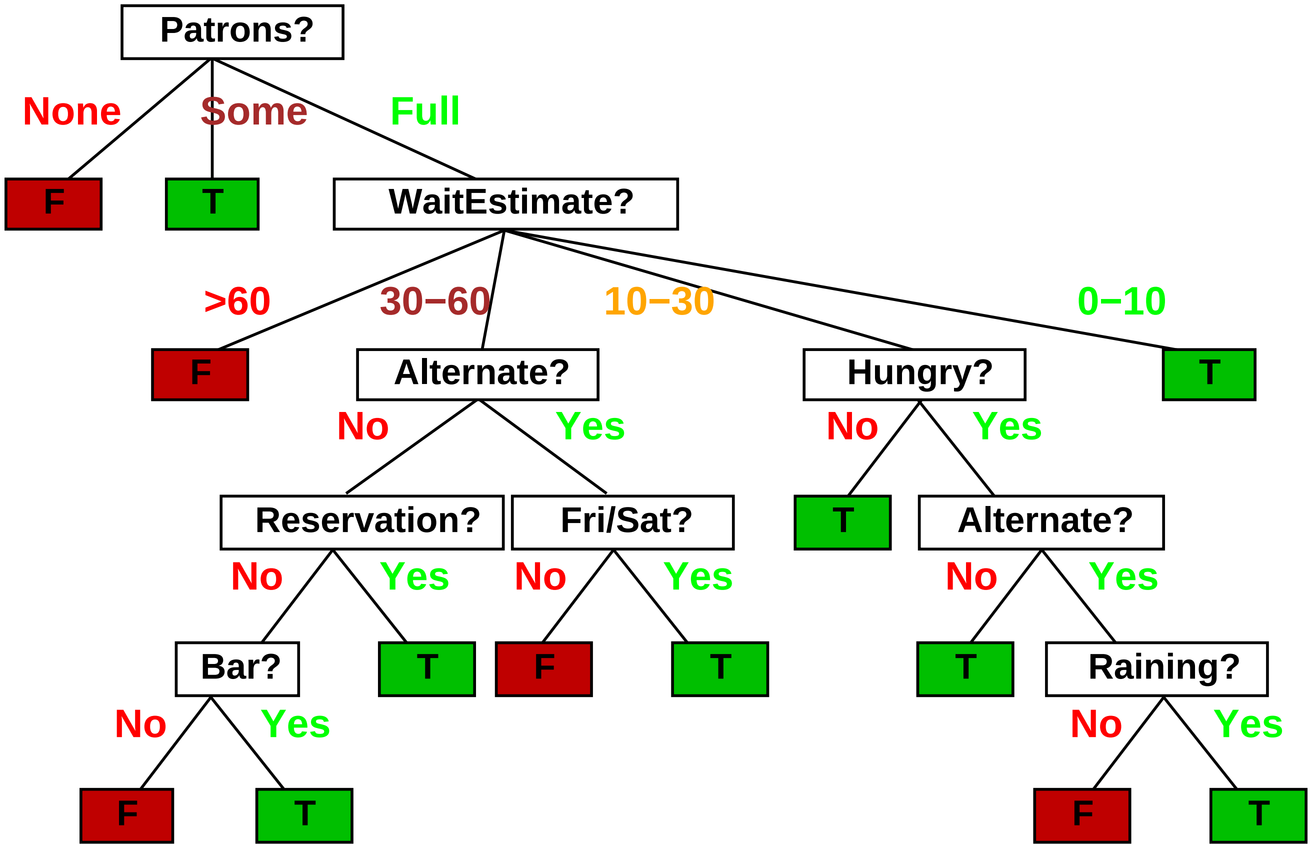
Example	Attributes										Target
	Alt	*Bar*	*Fri*	*Hun*	*Pat*	*Price*	*Rain*	*Res*	*Type*	*Est*	*WillWait*
X_1	*T*	*F*	*F*	*T*	*Some*	*$$$*	*F*	*T*	*French*	*0–10*	*T*
X_2	*T*	*F*	*F*	*T*	*Full*	*$*	*F*	*F*	*Thai*	*30–60*	*F*
X_3	*F*	*T*	*F*	*F*	*Some*	*$*	*F*	*F*	*Burger*	*0–10*	*T*
X_4	*T*	*F*	*T*	*T*	*Full*	*$*	*F*	*F*	*Thai*	*10–30*	*T*
X_5	*T*	*F*	*T*	*F*	*Full*	*$$$*	*F*	*T*	*French*	*>60*	*F*
X_6	*F*	*T*	*F*	*T*	*Some*	*$$*	*T*	*T*	*Italian*	*0–10*	*T*
X_7	*F*	*T*	*F*	*F*	*None*	*$*	*T*	*F*	*Burger*	*0–10*	*F*
X_8	*F*	*F*	*F*	*T*	*Some*	*$$*	*T*	*T*	*Thai*	*0–10*	*T*
X_9	*F*	*T*	*T*	*F*	*Full*	*$*	*T*	*F*	*Burger*	*>60*	*F*
X_{10}	*T*	*T*	*T*	*T*	*Full*	*$$$*	*F*	*T*	*Italian*	*10–30*	*F*
X_{11}	*F*	*F*	*F*	*F*	*None*	*$*	*F*	*F*	*Thai*	*0–10*	*F*
X_{12}	*T*	*T*	*T*	*T*	*Full*	*$*	*F*	*F*	*Burger*	*30–60*	*T*

Classification of examples is positive (T) or negative (F)

Decision trees

One possible representation for hypotheses

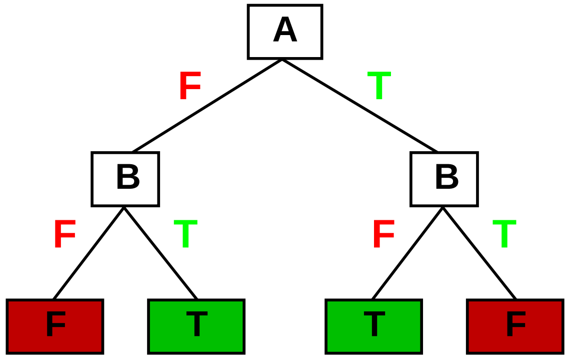
E.g., here is the "true" tree for deciding whether to wait:



Expressiveness

Decision trees can express any function of the input attributes.
E.g., for Boolean functions, truth table row $\rightarrow$ path to leaf:

A	B	A xor B
F	F	F
F	T	T
T	F	T
T	T	F



Trivially, $\exists$ a consistent decision tree for any training set w/ one path to leaf for each example (unless f nondeterministic in x) but it probably won't generalize to new examples

Prefer to find more **compact** decision trees

Hypothesis spaces

How many distinct decision trees with n Boolean attributes??

Hypothesis spaces

How many distinct decision trees with n Boolean attributes??

$=$ number of Boolean functions

Hypothesis spaces

How many distinct decision trees with n Boolean attributes??

$=$ number of Boolean functions
$=$ number of distinct truth tables with 2^n rows

Hypothesis spaces

How many distinct decision trees with n Boolean attributes??

$=$ number of Boolean functions
$=$ number of distinct truth tables with 2^n rows $= 2^{2^n}$

Hypothesis spaces

How many distinct decision trees with n Boolean attributes??

= number of Boolean functions
= number of distinct truth tables with 2^n rows = 2^{2^n}

E.g., with 6 Boolean attributes, there are 18,446,744,073,709,551,616 trees

Hypothesis spaces

How many distinct decision trees with n Boolean attributes??

= number of Boolean functions
= number of distinct truth tables with 2^n rows $= 2^{2^n}$

E.g., with 6 Boolean attributes, there are 18,446,744,073,709,551,616 trees

How many purely conjunctive hypotheses (e.g., $Hungry \land \neg Rain$)??

Hypothesis spaces

How many distinct decision trees with n Boolean attributes??

$=$ number of Boolean functions
$=$ number of distinct truth tables with 2^n rows $= 2^{2^n}$

E.g., with 6 Boolean attributes, there are 18,446,744,073,709,551,616 trees

How many purely conjunctive hypotheses (e.g., $Hungry \land \neg Rain$)??

Each attribute can be in (positive), in (negative), or out
$\Rightarrow$ 3^n distinct conjunctive hypotheses

More expressive hypothesis space
– increases chance that target function can be expressed
– increases number of hypotheses consistent w/ training set
$\Rightarrow$ may get worse predictions

Decision tree learning

Aim: find a small tree consistent with the training examples

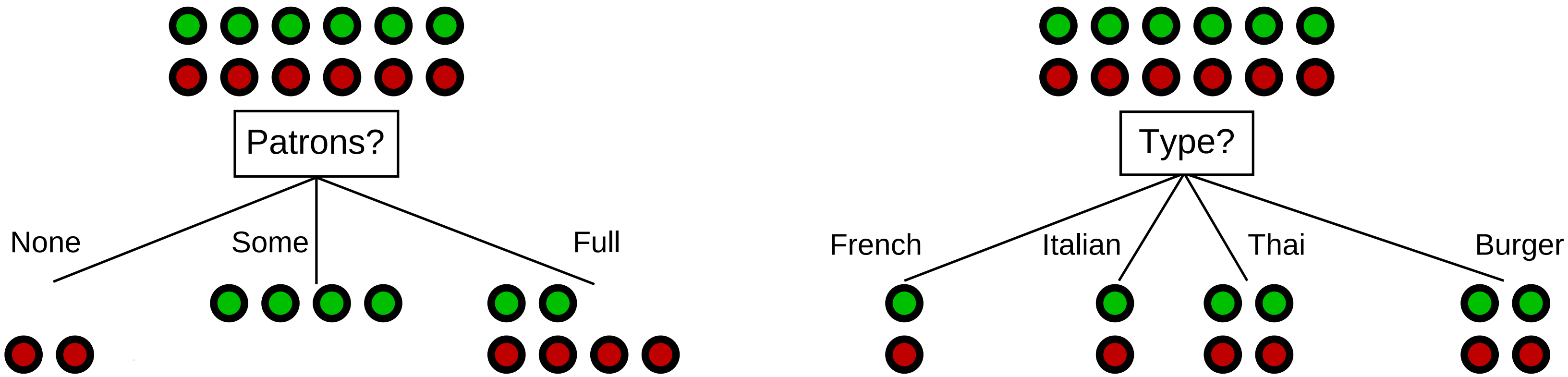
Idea: (recursively) choose "most significant" attribute as root of (sub)tree

function DTL(*examples, attributes, default*) **returns** a decision tree

    if *examples* is empty **then return** *default*
    else if all *examples* have the same classification **then return** the classification
    else if *attributes* is empty **then return** MODE(*examples*)
    else
        best ← CHOOSE-ATTRIBUTE(*attributes*, *examples*)
        tree ← a new decision tree with root test *best*
        for each value v_i of *best* **do**
            $examples_i$ ← {elements of *examples* with $best = v_i$}
            subtree ← DTL($examples_i$, $attributes - best$, MODE(*examples*))
            add a branch to *tree* with label v_i and subtree *subtree*
        return *tree*

Choosing an attribute

Idea: a good attribute splits the examples into subsets that are (ideally) "all positive" or "all negative"



Patrons? is a better choice—gives **information** about the classification

Information

Information answers questions

The more clueless I am about the answer initially, the more information is contained in the answer

Scale: 1 bit = answer to Boolean question with prior $\langle 0.5, 0.5 \rangle$

Information in an answer when prior is $\langle P_1, \ldots, P_n \rangle$ is

$$H(\langle P_1, \ldots, P_n \rangle) = \sum_{i=1}^{n} - P_i \log_2 P_i$$

(also called entropy of the prior)

Information contd.

Suppose we have p positive and n negative examples at the root
$\Rightarrow$ $H(\langle p/(p+n), n/(p+n)\rangle)$ bits needed to classify a new example
E.g., for 12 restaurant examples, $p=n=6$ so we need 1 bit

An attribute splits the examples E into subsets E_i, each of which (we hope) needs less information to complete the classification

Let E_i have p_i positive and n_i negative examples
$\Rightarrow$ $H(\langle p_i/(p_i+n_i), n_i/(p_i+n_i)\rangle)$ bits needed to classify a new example
$\Rightarrow$ **expected** number of bits per example over all branches is

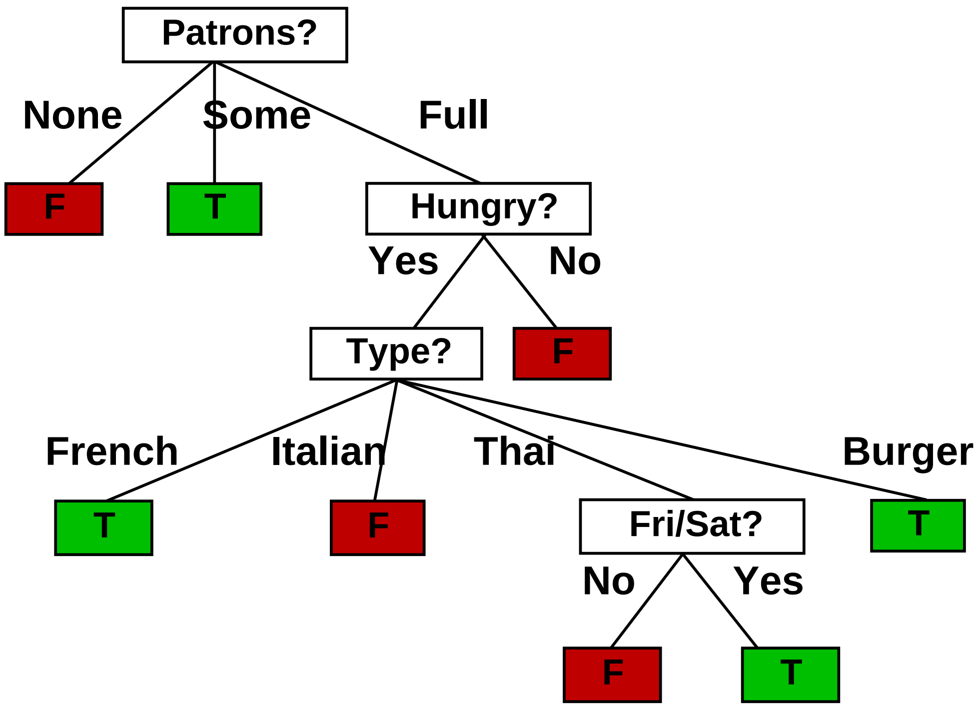
$$\Sigma_i \; \frac{p_i + n_i}{p+n} \, H(\langle p_i/(p_i + n_i), n_i/(p_i + n_i)\rangle)$$

For $Patrons?$, this is 0.459 bits, for $Type$ this is (still) 1 bit

$\Rightarrow$ choose the attribute that minimizes the remaining information needed

Example contd.

Decision tree learned from the 12 examples:



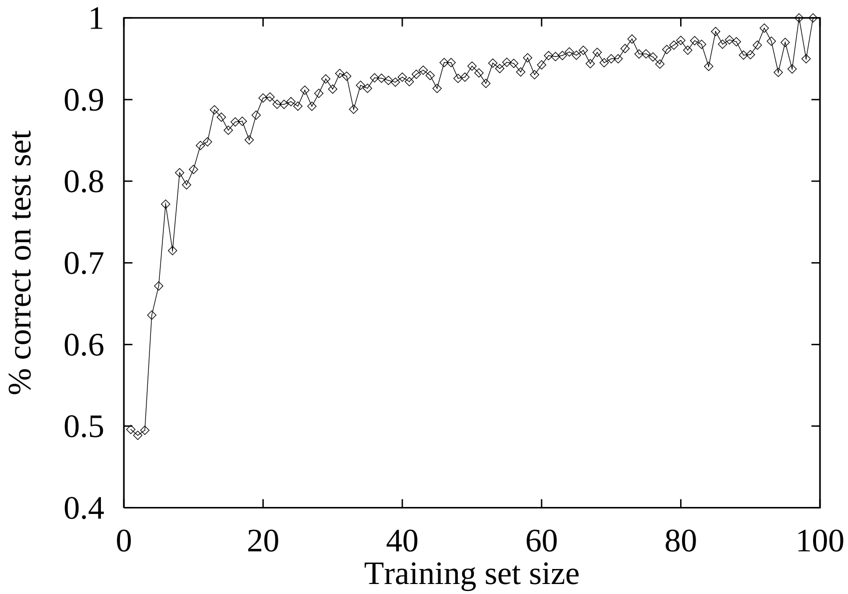
Substantially simpler than "true" tree—a more complex hypothesis isn't justified by small amount of data

Performance measurement

How do we know that $h \approx f$? (Hume's **Problem of Induction**)

Try h on a new test set of examples
(use **same distribution over example space** as training set)

Learning curve = % correct on test set as a function of training set size



Summary

Learning needed for unknown environments, lazy designers

For supervised learning, the aim is to find a simple hypothesis approximately consistent with training examples

Decision tree learning using information gain

Learning performance = prediction accuracy measured on test set