The University of Waikato
Department of Computer Science
Dr Tim Stokes and Dr Robi Malik

COMP 340-08B

Reasoning about Programs

Assignment 11

Exercise 1 (6 marks)

Prove using Hoare logic that the Hoare triple

$$(\!|a = 1|\!)\ P\ (\!|a = r^n|\!)$$

is totally correct, where P is the following piece of code.

```
i := 1;
while (i != n+1) {
  a := a * r;
  i := i + 1;
}
```

Exercise 2 (4+1+4 marks)

Use program algebra to prove the following.

a) For tests α, β and programs p, q,

$$\alpha[\beta[p, q], \beta[r, s]] = \beta[\alpha[p, r], \alpha[q, s]]\ .$$

b) For any test α, $(\!|\alpha|\!)\ 1\ (\!|\alpha|\!)$ is partially correct (where 1 denotes "skip").

c) The "disjunction rule" is sound:

$$\frac{(\!|\alpha_1|\!)\ p\ (\!|\beta_1|\!), (\!|\alpha_2|\!)\ p\ (\!|\beta_2|\!)}{(\!|\alpha_1 \vee \alpha_2|\!)\ p\ (\!|\beta_1 \vee \beta_2|\!)}\ .$$

Hint. In Boolean algebra, $\alpha \wedge (\alpha \vee \beta) = \alpha \vee (0 \wedge \beta) = \alpha \vee 0 = \alpha$.

Exercise 3   (5+10 marks)

Annotate the following Java program with JML assertions and prove its partial correctness.

```
/*@
  @ requires a != null;
  @ requires (\forall int i; i > 0 && i < a.length; a[i-1] <= a[i]);
  @ ensures \result >= 0;
  @ ensures \result <= a.length;
  @ ensures (\forall int i; i >= 0 && i < \result; a[i] < item);
  @ ensures (\forall int i; i >= \result && i < a.length; item <= a[i]);
  @ ensures (\exists int i; i >= 0 && i < a.length; a[i] == item) ==>
  @          a[\result] == item;
  @*/
public static int binarySearch(final int[] a, final int item)
{
  int lower = 0;
  int upper = a.length;
  while (lower < upper) {
    final int mid = (upper + lower) / 2;
    if (item <= a[mid]) {
      upper = mid;
    } else {
      lower = mid + 1;
    }
  }
  return lower;
}
```

A Java source file `BinarySearch.java` containing this method and some test code can be downloaded from the course home page in Moodle.

a) Annotate the `binarySearch()` method with so many assertions as are needed to completely prove that the postconditions are satisfied in all cases, and confirm the correctness of your assertions using the JML tools. *Please include a printout of your annotated program code with your submission.*

b) Extract all the predicate logic formulas that have to be proved to establish partial correctness of the `binarySearch()` method using Hoare logic, and sketch their proofs on paper.

Submission

Please put your written answers to the exercises and a printout your annotated program code for exercise 3 in the COMP340 slot on level 3 of G Block. In addition, please submit your annotated Java file electronically through the COMP 340-08B course home page in Moodle before the due date.

Due date: Wednesday 15 October 2008, 17:00