The University of Waikato
Department of Computer Science
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COMP 340-08B

Reasoning about Programs

Assignment 4

Exercise 1 (5 marks)

Download the J∀P∃ theory file `comp340-ass4.jt` from the COMP 340-08B course home page in Moodle at

`http://elearn.waikato.ac.nz/course/view.php?id=2567`

and use J∀P∃ to prove all the conjectures in it.

Exercise 2 (1+1 marks)

The following proofs in the system of natural deduction are both incorrect. Please explain which lines are incorrect, and why they are incorrect.

a)

1:	$\exists x$ brillig(x)	premise
2:	$\forall x$ (brillig(x) $\rightarrow$ (slithy(x) $\vee$ tove(x)))	premise
3:	$\forall x$ (slithy(x) $\rightarrow$ mimsy(x))	premise
4:	brillig(c)	assumption
5:	brillig(c) $\rightarrow$ (slithy(c) $\vee$ tove(c))	$\forall$-elim 2
6:	slithy(c) $\vee$ tove(c)	$\rightarrow$-elim 5,4
7:	slithy(c)	$\vee$-elim 6
8:	slithy(c) $\rightarrow$ mimsy(c)	$\forall$-elim 3
9:	mimsy(c)	$\rightarrow$-elim 8,7
10:	$\exists x$ mimsy(x)	$\exists$-intro 9
11:	$\exists x$ mimsy(x)	$\exists$-elim 1,4–10

b)

1:	$\exists x$ brillig(x) $\wedge$ $\exists x$ tove(x)	premise
2:	$\forall x$ (brillig(x) $\wedge$ tove(x) $\rightarrow$ mimsy(x))	premise
3:	$\forall x$ (mimsy(x) $\rightarrow$ slithy(x))	premise
4:	brillig(c) $\wedge$ tove(c)	assumption
5:	brillig(c) $\wedge$ tove(c) $\rightarrow$ mimsy(c)	$\forall$-elim 2
6:	mimsy(c)	$\rightarrow$-elim 5,4
7:	mimsy(c) $\rightarrow$ slithy(c)	$\forall$-elim 3
8:	slithy(c)	$\rightarrow$-elim 7,6
9:	$\exists x$ slithy(x)	$\exists$-intro 8
10:	$\exists x$ slithy(x)	$\exists$-elim 1,4-9

Exercise 3 (2+2+3+3+3 marks)

Show that the following arguments are correct in predicate logic, using the system of natural deduction. Write your proofs on paper, listing each proof step in a numbered line of its own, and indicating for each line how it is obtained. Use boxes to indicate the scope of assumptions and introduced constants.

a) **Premise 1:** $\mathsf{tove}(\mathsf{alice}) \wedge \mathsf{mimsy}(\mathsf{alice})$
Premise 2: $\forall x\ (\mathsf{tove}(x) \rightarrow \mathsf{slithy}(x))$
Premise 3: $\neg\exists x\ (\mathsf{mimsy}(x) \wedge \mathsf{slithy}(x))$
Conclusion: *false*

b) **Premise 1:** $\forall x\ ((\mathsf{brillig}(x) \vee \mathsf{tove}(x)) \rightarrow \mathsf{mimsy}(x))$
Premise 2: $\forall x\ ((\mathsf{slithy}(x) \vee \mathsf{mimsy}(x)) \rightarrow \mathsf{tove}(x))$
Premise 3: $\exists x\ \mathsf{slithy}(x)$
Conclusion: $\exists x\ \mathsf{mimsy}(x)$

c) **Premise 1:** $\forall x\ (\mathsf{brillig}(x) \rightarrow (\mathsf{mimsy}(x) \vee \mathsf{slithy}(x)))$
Premise 2: $\forall x\ (\neg\,\mathsf{slithy}(x) \rightarrow \neg\,\mathsf{mimsy}(x))$
Premise 3: $\forall x\ (\mathsf{slithy}(x) \rightarrow \mathsf{tove}(x))$
Conclusion: $\forall x\ (\mathsf{brillig}(x) \rightarrow \mathsf{tove}(x))$

d) **Premise 1:** $\mathsf{wabe}(\mathsf{alice})$
Premise 2: $\forall x\ \mathsf{mimsy}(x)$
Premise 3: $\forall x\ ((\mathsf{brillig}(x) \vee \mathsf{tove}(x)) \rightarrow \exists y\ (\mathsf{gyre}(x,y) \wedge \mathsf{slithy}(y)))$
Premise 4: $\forall x\ (\mathsf{mimsy}(x) \rightarrow \mathsf{brillig}(x))$
Conclusion: $\exists x\,\exists y\ (\mathsf{gyre}(x,y) \wedge \mathsf{brillig}(y))$

e) **Premise:** $\forall x\,\forall y\ (\mathsf{kiss}(x,y) \wedge \neg\,\mathsf{frog}(x) \leftrightarrow \mathsf{saved}(y))$
Conclusion: $\forall x\ \mathsf{frog}(x) \rightarrow \neg\exists y\ \mathsf{saved}(y)$

Submission

Please put your written answers to exercises 2 and 3 into the box marked **COMP340** in front of room G 1.15 before the due date.

Please save your J∀P∃ proofs for exercise 1 and submit them electronically through the COMP 340-08B course home page in Moodle at

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before the due date.

Due date: Wednesday, 13th August 2008, 17:00

'Twas brillig, and the slithy toves
Did gyre and gimble in the wabe.
All mimsy were the borogroves,
And the mome raths outgrabe.
...

"Through the Looking-Glass" by Lewis Carroll