COMP340B: Logic and Computation

Assignment 7

Due 5PM, Wednesday 17 September, 2008

Note: please submit your assignment to the COMP340 slot on Level 3 of G Block. Please submit by the due time, or expect to incur the 10% *late penalty (going up* 10% *for each additional day late). (*Note the change of place to hand in!!!*)*

1. [2 marks] For the code below, suppose the initial state is $(x, y, z) = (1, 2, 3)$. Find the state after each assignment.

$$z := x + y;\ z := z * z;\ z := 2 * z$$

2. [7 marks] Consider the piece of code below.

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if (m>n) then
    {while (m>n) {m:=m-1} }
         else
    {while (m<n) {n:=n-1} }
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   (a) Suppose the initial state is $(m, n) = (5, 2)$. Describe in words exactly what the piece of code does to this state vector. Do not just list the way it changes: explain what is going on.

   (b) What does the code compute, given any state vector $(m, n) \in \mathbb{N} \times \mathbb{N}$? Does it always terminate?

Please turn over!

3. [2 marks] A program construct used by some languages is **repeat-until**. For a piece of code P and a test α, define

$$\text{repeat } \{P\} \text{ until } (\alpha)$$

to be the program which

- applies P to a given state vector;
- applies α to the result: if the result is *false*, it resumes with the first step, and otherwise terminates.

Show how to define "repeat $\{P\}$ until (α)" in terms of only the constructs discussed in the lectures.

4. [6 marks] Given that the following Hoare triple is (partially) correct,

$$(\!|x \geq 1|\!)\ C\ (\!|y \geq 1|\!),$$

determine which of the following are definitely correct. For each of those which is, give an argument why, and for each of those which is not, give an example of a piece of code C for which the above triple is correct but the one given in the question is not. Do not use any Hoare Logic rules, just use the definition of partial correctness.

(a) $(\!|x > 1|\!)\ C\ (\!|y \geq 1|\!)$

(b) $(\!|x > 1|\!)\ C\ (\!|y > 1|\!)$

(c) $(\!|x = 1|\!)\ C\ (\!|y > 0|\!)$

5. [8 marks] For each Hoare triple below, find the weakest possible precondition given the postcondition and piece of code. Hence decide which of the triples are correct.

(a) $(\!|a > 3|\!)\ a := a * a\ (\!|a > 9|\!)$

(b) $(\!|a < b|\!)\ a := b\ (\!|a < b|\!)$

(c) $(\!|\ |\!)\ a := b - c\ (\!|a > 0|\!)$

6. [5 marks] "When the Assignment Rule is applied to the piece of code $y := 1/x$ with postcondition $(\!|xy = 1|\!)$, one can show that the following is a totally correct Hoare triple:

$$(\!|\ |\!)\ y := 1/x\ (\!|xy = 1|\!).$$

Yet this is not totally correct because $1/x$ is not even defined unless $x \neq 0$. Somehow the Assignment Rule misses out this needed precondition."

Discuss this apparent problem with the Assignment Rule: is it a real problem with the rule itself?