COMP340B: Reasoning about Programs

Assignment 8

Due 5PM, Wednesday 24 September, 2008

Note: please submit your assignment to the COMP340 slot on Level 3 of G Block. Please submit by the due time, or expect to incur the 10% *late penalty (going up* 10% *for each additional day late).*

1. [4 marks] The following code has the postcondition $(\!|\ x + by = u,\ cy - 2x = 0\ |\!)$. (Here, x, b, y, u, c are real variables.) Find the weakest possible precondition.

```
y:= 2/(2*b+c); x:=1-b*y
```

2. [8 marks] Suppose that the following triple is partially correct. (Here, the state vector $(x, y, u, v, \ldots)$ has all variables of real type.)

   $(\!|x < y|\!)$ P $(\!|u < v|\!)$

   Decide which of the following triples are correct. For those that are, give a proof using relevant rules. For those that are not, give a counterexample (that is, find a piece of code P for which the above triple is correct but the one in the question is not). There is no need to prove your counterexample is a counterexample, but at least give one.

   (a) $(\!|x \leq y|\!)$ P $(\!|u < v|\!)$

   (b) $(\!|x \leq y - 2|\!)$ P $(\!|u < v|\!)$

   (c) $(\!|x \leq y|\!)$ P $(\!|u \leq v|\!)$

   (d) $(\!|x < y|\!)$ P $(\!|u \leq v|\!)$

Please turn over!

3. [9 marks] Prove that the following Hoare triples are correct. (Assume all variables are real.)

   (a) $(\!|\ |\!)$ **if** $(a > 0)$ **then** $\{x := a\}$ **else** $\{x := -a\}$ $(\!|x^2 = a^2|\!)$

   (b) $(\!|x * y = 0|\!)$ **if** $(x = 0)$ **then** $\{y := x\}$ $(\!|y = 0|\!)$

4. [9 marks] Prove that the piece of code below calculates the factorial function

$$n! \ = \ 1 \cdot 2 \cdot \ldots \cdot n,$$

   where we define $0! = 1$.

```
prod:=1;
i:=1;
while (i != n+1)
     { prod:=prod*i;
       i:=i+1  }
```

   (This means you need to show that the code gives a partially correct Hoare triple when appropriate choices of pre- and postconditions are made.)