COMP340-08B Reasoning About Programs

1. Propositional Logic

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COMP 340-08B Lectures and Tutorials

Lecture Tue 15:10 G 3.33

Lecture Wed 12:00 G B.13

Lecture Thu 13:10 G 3.33

Tutorial Fri 9:00 K G.06 or R G.19

Assessment

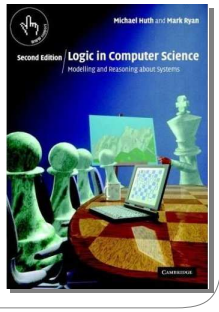
Internal Assessment: 50%

- 11 assignments handed out in Thursday lectures, due Wednesday 17:00 the following week.
- Assignments 1–5 and 7–11 are worth 1/12 of internal assessment.
- Assignment 6 is worth 2/12 of internal assessment.

Final Exam: 50%

COMP340-08B Textbook

Michael Huth and Mark Ryan, *Logic in Computer Science*. 2nd edition, Cambridge University Press, 2004.



Logic

'Contrariwise,' continued Tweedledee, 'If it was so, it might be; and if it were so, it would be: but as it isn't, it ain't. That's logic.'

— Lewis Carroll

Why Study Logic?

Rules of logic used in many areas of computer science, for example:

- Programming (e.g. && and || in C)
- Circuit design
- Artificial intelligence
- Proving the correctness of a program

Euclidean Algorithm

```
public int gcd(int x, int y)
{
  if (y == 0) {
    return x;
  } else {
    return gcd(y, x % y);
  }
}
```


COMP340-08B Prerequisites

- **COMP103**
  Introduction to Computer Science I
- **COMP153**
  Practical Programming
- **COMP140**
  Foundations of Computer Science
- **COMP235**
  Logic and Computation
- **COMP240**
  Mathematical Foundations of Computer Science

Logic Symbols

$\neg \quad \wedge \quad \vee$

Logic Symbols

$\rightarrow$

On the Nature of Implication

Let:

p = “The moon is made of green cheese.”

q = “The lecturer is a pink elephant.”

What is the truth value of …

$$p \rightarrow q$$

Logic Symbols

$\leftrightarrow \quad \oplus$

Propositional Connectives

Negation	$\neg p$	"not p"
Conjunction	$p \wedge q$	"p and q"
Disjunction	$p \vee q$	"p or q"
Exclusive Or	$p \oplus q$	"either p or q"
Implication	$p \rightarrow q$	"if p then q"
Equivalence	$p \leftrightarrow q$	"p if and only if q"

Logic Symbols

$\forall \quad \exists$

Propositional vs. Predicate Logic

Propositional Logic

- Only propositions and connectives.

$\neg \quad \wedge \quad \vee \quad \rightarrow \quad \leftrightarrow \quad \oplus$

Predicate Logic

- Adds variables and quantifiers.

$\forall \quad \exists$

Logic Concepts

- Formula
- Model
- Tautology
- De Morgan's Law
- Karnaugh Map
- Logical Consequence
- Proof
- Induction
- Deduction
- Soundness
- Completeness
- Proposition

What is a Proposition?

Definition:

A **proposition** is a statement that is either true or false.

Example:

"Hamilton is south of Auckland."

Propositional Variables

- **Propositional variables** are the basic components of propositional logic.
- Written as p, q, r, …
- They are placeholders for propositions.
- **Example:**
  p = "The fridge is empty."

Truth Values

Every propositional variable has a **truth value** attached to it, which is either

- **T** – true, or
- **F** – false.

Example:

p = "The fridge is empty."

- If the fridge is empty then p has value **T**.
- Otherwise p has value **F**.

Constructing Formulas

- Building complex propositions from simpler ones:
  - Propositional variables are joined together using **logical connectives**.
- The complex propositions are called **formulas** (or **sentences**).

Some Formulas

$$\neg\neg\neg p$$

$$(p \wedge\wedge\, q)$$

$$(((p \wedge q) \oplus r) \wedge s)$$

$$((p \wedge q) \rightarrow (q \wedge p))$$

$$(p \wedge ((q \leftrightarrow r) \wedge \neg p))$$

Evaluating Formulas

If we know the truth value of each basic proposition, we can already determine the truth value of a formula automatically!

Example:

"Hamilton is south of Auckland." **and** *"Auckland is east of Sydney."*

Truth Tables of the Connectives

p	q	$\neg p$	$p \wedge q$	$p \vee q$	$p \rightarrow q$	$p \leftrightarrow q$	$p \oplus q$
F	F						
F	T						
T	F						
T	T						