COMP340-08B Reasoning About Programs

3. Logical Arguments

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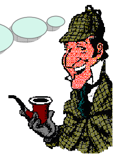
Deriving New Knowledge

Assume we know certain things to be true …

$1 + 1 = 2$

$1 + 2 = 3$

$x + y = y + x$



What other things can we conclude?

Rational Arguments

When does one sentence follow logically from some others?

Let's consider the structure of an argument:

If Socrates is a man then Socrates is mortal. Socrates is a man.	*Premises*
Therefore, Socrates is mortal.	*Conclusion*

Does this Argument Make Sense?

If Socrates is a man then Socrates is mortal. Socrates is a man.	*Premises*
Therefore, Socrates is mortal.	*Conclusion*

Let's formalize the argument:

p = "Socrates is a man" q = "Socrates is mortal"

$p \to q$ p	*Premises*
q	*Conclusion*

Correct Arguments

$p \to q$ p	*Premises*
q	*Conclusion*

Definition:

An argument is **correct** if the conclusion is a logical consequence of the premises.

Checking the Argument

Checking whether:

q is a logical consequence of $\Phi = \{p, p \to q\}$.

p	q	p	$p \to q$	q
F	**F**	**F**	**T**	**F**
F	**T**	**F**	**T**	**T**
T	**F**	**T**	**F**	**F**
T	**T**	**T**	**T**	**T**

Remember …

To check whether A is a logical consequence of Φ:

1. Construct truth tables for Φ and A.
2. If every line showing the truth value **T** for all formulas in Φ also shows the truth value **T** for A, then A is a logical consequence of Φ.
3. Lines showing the truth value **F** for some formula in Φ do not matter.

An Incorrect Argument

Socrates is at home or Socrates is happy.
Socrates is happy. } *Premises*

Therefore, Socrates is at home. } *Conclusion*

Let's formalize the argument:

p = "Socrates is at home" q = "Socrates is happy"

$p \vee q$
q } *Premises*

p } *Conclusion*

Checking the Argument

Checking whether:

p is a logical consequence of $\Phi = \{ p \vee q, q \}$.

p	q	$p \vee q$	q	p
F	F	F	F	F
F	T	T	T	F
T	F	T	F	T
T	T	T	T	T

Another Argument

Socrates is happy.
Socrates is not happy. } *Premises*

Therefore, Socrates is at home. } *Conclusion*

Let's formalize the argument:

p = "Socrates is at home" q = "Socrates is happy"

q
$\neg q$ } *Premises*

p } *Conclusion*

Checking the Argument

Checking whether:

p is a logical consequence of $\Phi = \{ \neg q, q \}$.

p	q	$\neg q$	q	p
F	F	T	F	F
F	T	F	T	F
T	F	T	F	T
T	T	F	T	T

Note

- If the premises of an argument are false, the conclusion does not matter!
- An argument with false premises is always correct.

Checking Arguments

- So far we have used truth tables to check whether an argument is correct.
- A different way of asserting the correctness of an argument is to find a **proof** for it.

What is a Proof?

A proof is a step-by-step demonstration that the conclusion follows from the premises.

In each step,
we are only allowed to use

- sound **rules of inference** or
- sound **laws of equivalence**.

Inference Rule: Modus Ponens

(**MP**) $$\frac{A \quad A \rightarrow B}{B}$$

Preconditions (above the line); *Immediate Conclusion* (below the line)

"From A and $A \rightarrow B$, infer B."

Sample proof:

1. "Socrates is a man"	**Premise**
2. "Socrates is a man" → "Socrates is mortal"	**Premise**
3. "Socrates is mortal"	**MP**(1,2)

Laws of Equivalence

Commutativity:
$A \wedge B$ is logically equivalent to $B \wedge A$
plus the same for the connectives $\vee$, $\oplus$, $\leftrightarrow$

Associativity:
$A \wedge (B \wedge C)$ is logically equivalent to $(A \wedge B) \wedge C$
plus the same for the connectives $\vee$, $\oplus$, $\leftrightarrow$

Distributivity:
$A \wedge (B \vee C)$ is equivalent to $(A \wedge B) \vee (A \wedge C)$
$A \vee (B \wedge C)$ is equivalent to $(A \vee B) \wedge (A \vee C)$

More Laws of Equivalence

Excluded Middle Law:
$A \wedge \neg A$ is logically equivalent to false
$A \vee \neg A$ is logically equivalent to true

Identity Laws:
$A \wedge \mathit{true}$ is logically equivalent to A
$A \vee \mathit{false}$ is logically equivalent to A

Domination Laws:
$A \wedge \mathit{false}$ is logically equivalent to false
$A \vee \mathit{true}$ is logically equivalent to true

More Laws of Equivalence

Double Negation Law:
A is logically equivalent to $\neg\neg A$

De Morgan's Laws:
$\neg(A \wedge B)$ is logically equivalent to $\neg A \vee \neg B$
$\neg(A \vee B)$ is logically equivalent to $\neg A \wedge \neg B$

Definition of $\rightarrow$:
$A \rightarrow B$ is equivalent to $\neg A \vee B$

Definition of $\leftrightarrow$:
$A \leftrightarrow B$ is equivalent to $(A \rightarrow B) \wedge (B \rightarrow A)$