COMP340-08B Reasoning About Programs

17. Resolution Theorem Proving

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Yet Another Deductive System

— Resolution —

- A very simple deductive system
- Easy to implement
- Used by automatic theorem provers
- First presented by Robinson (1965)

The Deductive System of Resolution

Axioms:

(none)

Rules of inference:

(**Res**) $$\frac{A \vee C \quad B \vee \neg C}{A \vee B}$$

How to Use Resolution

To determine whether some well-formed formula A is ***unsatisfiable****:*

1. Convert A to Conjunctive Normal Form (CNF).
2. If resolution produces the empty disjunction, i.e., *false*, then A is unsatisfiable.

Clausal Form

Clausal form is obtained from Conjunctive Normal Form by writing every disjunction of literals, i.e., every **clause**

$$L_1 \vee L_2 \vee \ldots \vee L_n$$

as a set

$$\{L_1, L_2, \ldots, L_n\}.$$

Ignores order of literals and removes duplicates.

The **empty clause**, i.e., *false* or $\{\}$, is usually written as $\square$.

Resolution Rule in Clausal Form

(Res) $$\frac{C_1 \cup \{p\} \quad C_2 \cup \{\neg p\}}{C_1 \cup C_2}$$

More Precisely …

To determine whether some well-formed formula A is **unsatisfiable**:

1. Convert A to Clausal Form.
2. If resolution produces the empty clause □, then A is unsatisfiable.

Some Fundamental Facts

Proposition

A well-formed formula A is **valid** if and only if $\neg A$ is **unsatisfiable**.

Proposition

A well-formed formula A is a **logical consequence** of a set of well-formed formulas Φ, if and only if $\Phi \cup \{\neg A\}$ is unsatisfiable.

More Uses of Resolution

To determine whether a formula A is **valid**:

1. Convert $\neg A$ to clausal form.
2. If resolution produces the empty clause □, then A is valid.

To determine whether a formula A is a **logical consequence** of a set of formulas Φ:

1. Convert all formulas in Φ to clauses.
2. Add the clauses obtained for $\neg A$.
3. If resolution produces the empty clause □, then A is a logical consequence of Φ.

Example

Prove that the following formulas are unsatisfiable together.

1. $\mathrm{unicorn(vera)} \wedge \mathrm{black(vera)}$
2. $\mathrm{unicorn(vera)} \rightarrow \mathrm{white(vera)}$
3. $\neg(\mathrm{white(vera)} \wedge \mathrm{black(vera)})$

Step 1: Translation to Clausal Form

1a. $\{\mathrm{unicorn(vera)}\}$

1b. $\{\mathrm{black(vera)}\}$

2. $\{\neg\mathrm{unicorn(vera)}, \mathrm{white(vera)}\}$

3. $\{\neg\mathrm{white(vera)}, \neg\mathrm{black(vera)}\}$

Step 2: Apply Resolution

1.	$\{\mathrm{unicorn(vera)}\}$	Premise
2.	$\{\mathrm{black(vera)}\}$	Premise
3.	$\{\neg\mathrm{unicorn(vera)}, \mathrm{white(vera)}\}$	Premise
4.	$\{\neg\mathrm{white(vera)}, \neg\mathrm{black(vera)}\}$	Premise
5.	$\{\mathrm{white(vera)}\}$	Res: 1,3
6.	$\{\neg\mathrm{black(vera)}\}$	Res: 5,4
7.	□	Res: 2,6

Another Example

Given premises

1. brillig → mimsy ∨ slithy
2. ¬slithy → ¬mimsy
3. slithy → tove

prove that the following is a logical consequence

4. brillig → tove

Step 1: Translation to Clausal Form

1.	{¬brillig, mimsy, slithy}	Premises
2.	{slithy, ¬mimsy}	
3.	{¬slithy, tove}	
4a.	{brillig}	Negated Conclusion
4b.	{¬tove}	

Step 2: Apply Resolution

1.	{¬brillig, mimsy, slithy}	Premise
2.	{slithy, ¬mimsy}	Premise
3.	{¬slithy, tove}	Premise
4.	{brillig}	Premise
5.	{¬tove}	Premise
6.	{mimsy, slithy}	Res: 4,1
7.	{slithy}	Res: 6,2
8.	{tove}	Res: 7,3
9.	□	Res: 8,5

Soundness of Resolution

Theorem

Let Ψ be a set of clauses.
If the empty clause □ can be proven from Ψ by resolution, then Ψ is **unsatisfiable**.

Completeness of Resolution

Resolution is refutation-complete.

Theorem

Let Ψ be a set of clauses.
If Ψ is **unsatisfiable**, then the empty clause □ can be proven from Ψ by resolution.