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COMP340-08B

Natural Deduction

(∧-intro) $\dfrac{A \quad B}{A \wedge B}$ (∧-elim-1) $\dfrac{A \wedge B}{A}$ (∧-elim-2) $\dfrac{A \wedge B}{B}$

(∨-intro-1) $\dfrac{A}{A \vee B}$ (∨-intro-2) $\dfrac{B}{A \vee B}$ (∨-elim) $\dfrac{A \vee B \quad \dfrac{A}{C} \quad \dfrac{B}{C}}{C}$

(→-intro) $\dfrac{\dfrac{A}{B}}{A \to B}$ (→-elim) $\dfrac{A \to B \quad A}{B}$

(↔-intro) $\dfrac{A \to B \quad B \to A}{A \leftrightarrow B}$ (↔-elim-1) $\dfrac{A \leftrightarrow B}{A \to B}$ (↔-elim-2) $\dfrac{A \leftrightarrow B}{B \to A}$

(*false*-intro) $\dfrac{A \quad \neg A}{\mathit{false}}$ (*false*-elim) $\dfrac{\dfrac{\neg A}{\mathit{false}}}{A}$ (¬-intro) $\dfrac{\dfrac{A}{\mathit{false}}}{\neg A}$

(∀-elim) $\dfrac{\forall x A(x)}{A(t)}$ Provided that t does not contain any variables.

(∀-intro) $\dfrac{\overline{A(c)}}{\forall x A(x)}$ Provided that c is e *new* constant symbol.

(∃-elim) $\dfrac{\exists x A(x) \quad \dfrac{A(c)}{B}}{B}$ Provided that c is e *new* constant symbol.

(∃-intro) $\dfrac{A(t)}{\exists x A(x)}$